H-INFINITY CONTROLLED ACTUATORS IN AUTOMOTIVE ACTIVE SUSPENSION SYSTEM

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ABSTRACT
In this paper active suspension with linear electric motor has been designed. The active suspension is often hardly rejected for its big energy consumption. This contribution deals with direct real-time energy control in suspension system. The linear electric motor is used as an actuator. This motor can in specific situation generate the energy for next use. The H-infinity controller has been developed to improve car behavior, in particular passenger comfort and car stability. The second objective of the controller is to control energy distribution. Our submission is mainly focused on vehicle suspension, but it can be generalized to any kind of active suspension.

INTRODUCTION
Nowadays increased attention has been focused on suspension innovations. This paper objective is to describe active suspension design with attention to energy distribution control. In the time of growing interest of renewable energy resources the minimization of energy consumption can be small contribution to better utilization of energy resources. Especially for the car application the energy consumption play important role of the design process.

MOTIVATION
In this paper, active suspension with linear electric motor and $H_\infty$ controller has been designed. The biggest disadvantage in most active suspension systems is energy demands, which hardly increase together with increasing interference with system. This drawback can be eliminated in case of linear electric motor, because there is a possibility to recuperate energy during special suspension movements and consequently store this energy and use it later, when it is necessary. In the next chapters the way how to control energy distribution has been described.

Naturally, any kind of suspension systems are designed to meet certain requirements. Two important requirements should be improved in suspension systems – disturbance absorbing (in case of vehicle: passenger comfort) and attenuation of disturbance transfer to floor (in case of vehicle: car handling). The first requirement can be characterized as attenuation of damped mass acceleration or as peak minimization of damped mass vertical displacement. Second requirement is characterized as attenuation of force acting on the floor or in the simple model car as attenuation of unsprung mass acceleration.

Effort devoted to passive suspension design is ineffective, because there is a contradiction between both requirements. The best result (in sense of requirements improvement) can be achieved by active suspension, which means that some additional force can act on system.

SUSPENSION SYNTHESIS MODEL
The H-infinity controller synthesis has been done using simple basic active suspension model. Such model consists of suspension spring, damper, linear electric motor as actuator, sprung mass, unsprung mass and spring connecting unsprung mass to floor (road). See Fig. 1. More complex model has been used for analysis and validation. The complex model description is behind the scope of this article and interested reader can find it in the [1].
Linear electric motor can transform electrical energy to straight motion of the rotor. In fact, linear motor works on the same basis as conventional electric motor, the only difference is that linear motor has unfolded windings to straight direction. It can be imagined as common rotary motor with diameter equal to infinity. Basic principle is shown in Fig. 2 (figure has been adapted from manufacturer spreadsheets). The same principle in larger scale is used for example in magnetic levitation trains.

In this paper another property of motor has been utilized. The motor is able to recuperate energy, thus motor can transform energy from straight motion to electrical energy. That means linear motor use vertical motions to produce electrical energy.

The objective of controller design is to find the balance between energy demands and supply. In other words to find the trade-off between performance and energy consumption.

All experiments have been done using tubulus linear electric motor TBX3810 (ThrustTube). Its important properties are:
- peak force 2027N
- peak current 21.8A
- continuous stall force 293.2N
- electrical time constant 1.26ms
- continuous working voltage 320V ac
- maximum phase temperature 100 °C

Linear Motor Implementation

The important question is if the linear electric motor model should be included in the synthesis model or if it should be used for simulations only. Of course, the linear part can be added to controller design process, that means linear model including only the electrical and mechanical time constant.

Let’s emphasize the advantages and disadvantages of the model inclusion. The advantage is that the closed-loop will have more information and then can achieve better results. Unfortunately there are disadvantages too: the first is the system rank and consequently controller order grows from 4 to 5. The second problem is that the D matrix of statespace description for motor model has no full rank and hence implementation functions are limited or complicated.

According to comparison of the advantages and disadvantages the linear model has not been included in the synthesis model. It is possible to leave the linear motor model and to suppose the linear dependence of the desired force? Yes. Both the mechanical and the electrical constants are very small – about 1 ms. Moreover the H-infinity control design is robust enough, which will be proved by the simulations and experiments.

Energy Balance

As mentioned above, linear electric motor is able to recuperate energy from system. Roughly speaking, when generated force has the same direction as suspension speed, then energy has to be supplied to the system. Otherwise energy can be recuperated and stored for next use.

In fact, there are some nonlinearities in recuperation and therefore dependency of energy budget is a little bit more complicated. 3-D graph of interesting area can be plotted from the force-velocity profile and mathematical model. The Fig. 3 shows how much energy can be obtained for given force and velocity. Zero power means that energy has to be supplied to the system.
This characteristic (Fig. 3) provides one of requirements on control system, because optimization objective is to maximize of recuperated energy, with some trade-off.

**CONTROLLER**

The controller for active suspension has been designed using H-infinity theory, therefore basic idea of this theory is presented. The standard H-infinity control scheme is plotted in the Fig. 4. When open loop transfer matrix from $u_1$ to $y_1$ is denoted $T_{y_1u_1}$, then a standard optimal H-infinity controller problem is to find all admissible controllers $K(s)$ such that $\|T_{y_1u_1}\|$ is minimal.

For interested readers, the detailed description of the H-infinity theory can be found for example in [2].

An H-infinity controller is computed as $T_{y_1u_1}$ norm minimization. So it is possible to shape a characteristic in an open loop to improve performance of the whole system.

For active suspension system the performance and robustness output should be weighted. The performance weighting has to include the passenger comfort and car stability (body speed, suspension displacement, actuator force, ...). For linear electric motor as an actuator the additional weight is added to control maximum force, energy consumption and robustness of the system.

**QUANTIFICATION**

Quantitative measures have to be defined to evaluate the results achieved by the closed loop system and to compare the active and passive systems.

First requirement is to improve car stability and road friendliness, which can be characterized as an attenuation of the tire pressure, or more precisely unsprung mass to road deviation. To get a measurable value the RMS function has been evaluated:

$$J_{\text{stab}} = \sqrt{\int_0^T (z_w - z_b)^2 \, dt}$$

Where $z_w$ is wheel displacement and $z_b$ is body displacement.

Second, passenger comfort should be improved. This property can be observed as sprung mass acceleration attenuation. Therefore the measured RMS value is as follows:

$$J_{\text{comf}} = \sqrt{\int_0^T G_w \ast \ddot{z}_b \, dt}$$

Where $z_b$ is body acceleration, $G_w$ is weighting function for human sensitivity to vibration and $\ast$ means convolution.

**ENERGY CONTROL PRINCIPLE**

At first the H-infinity controller is designed using appropriate weights. The controller is optimized to minimum energy consumption. This optimality has to be defined as a trade-off between performance and energy consumption.

In the car the working conditions change according to drive situation. Hence it is very difficult to say globally what level of performance is enough and how much energy can be supplied. It would be ideal if there exist some possibility to control the energy consumption in a run-time. The energy should be controlled by external signal and depends on car and road conditions, i.e. on the energy store (battery) capacity and on the road surface.

One possibility is to analyze the drive conditions and recomputed the control in the real-time. This is very problematic, because the H-$\infty$ controller design is time consuming (where sample time has to be at least 1 ms!) and moreover the performance of the H-infinity controller cannot be guaranteed in all conditions. This is the reason why another solution has been chosen.

The resulted H-infinity controller is very robust and the active suspension system is relatively stable. Then it had been decided to control the energy consumption by controller deterioration. Shortly speaking lets assume two driving conditions:

- the car is driving through very bad terrain and there is enough energy stored in the battery system – then controller works in conventional mode, motor need the energy supply and suspension performance is preserved.
- the car is driving common road and there is not enough energy stored in the battery system, because the car drive through bad terrain before.
External signal gives the information to $H_\infty$ controller to deteriorate performance and to reduce the energy consumption. This deterioration is made by the desired force attenuation.

If the force is attenuated too much, then active suspension works almost as passive suspension and linear electric motor works in generator mode and produce energy for battery. Of course suspension performance is deteriorated (maximally on the passive suspension level). The influences of these controller modifications to suspension system performance are discussed in the following section.

The principle of proposed energy control is illustrated in the Fig. 5. The $H$-infinity controller is extended with the variable gain block controlled by external signal (energy control input).

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**ENERGY CONTROL ANALYSIS**

In the section V-A the energy control has been discussed as an extension of the $H_\infty$ controller abilities. Now the influence to the performance and robustness is presented. The $H$-infinity controller is deteriorated by desired force attenuation using input coefficient, which is given by the superior controller.

At first the robustness tests have to be done to find out the range of the input coefficient in the energy control block. The direct numerical method has been selected for robustness test, because the rank of the closed system is relative small (4 for plant + 6 for weights = 10 for system, 10 for system + 10 for controller = 20 total closed loop rank). Hence the poles have been tested for stability for given input coefficient. Moreover this characteristic is plotted to make graphical validation.

The graphical representation of the stability test is shown in the Fig. 6 and Fig. 7. The closed loop poles are plotted in the figures, where the input coefficient differ in the range from $-0.5$ to 1.7. The stars show the original $H$-infinity pole placement. The Fig. 7 is zoom of the Fig. 6 near the stability region.

The maximum and minimum stable input coefficient is given by the previous test. As a result the input coefficient has to lie between 0.000 and 1.613 to achieve stability.

The final coefficient range should be determined to achieve certain robustness. The resulted poles region with relative damping 1.4 and maximum real part $-0.1$ has been chosen as a condition. Selected region is plotted by the dashed line in the Fig. 7. According to modification principle (see previous section) it gives no sense to set the input coefficient greater than one. In conclusion the input coefficient range which satisfies defined condition is following:

- minimum: 0.512
- maximum: 1.000

Last the influence of input coefficient to active suspension performance is examined. The quantitatively measures has been compared using passive suspension performance. The random
road disturbance has been used as an first test input and the driving through bump as an second input. The comparison of different input coefficients (minimum and maximum) and its influence to active suspension system performance is summarized in the Table I. The road disturbance has been used as a plant input. The percentage are computed as relative improvement of active system against the passive suspension.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>0.512</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-infinity norm</td>
<td>0.455</td>
<td>0.359</td>
</tr>
<tr>
<td>Comfort</td>
<td>20.13%</td>
<td>29.89%</td>
</tr>
<tr>
<td>Stability</td>
<td>8.92%</td>
<td>12.83%</td>
</tr>
<tr>
<td>Energy</td>
<td>-71.1 J</td>
<td>127.6 J</td>
</tr>
</tbody>
</table>

Table I – Influence of input coefficient to performance

The quiet assumption has been made that controller improvements can be designed separately (against H-infinity controller). The previous table shows that controller modification can be designed with this assumption without dramatical lost of performance.

CONCLUSION

In this paper the $H_\infty$ controller for active suspension with linear electric motor has been designed. The method for direct real-time energy control has been proposed with respect to lower the energy consumption of the closed loop system. The method modifies the standard H-infinity controller and develop stable controller with variable energy demands. The results has been validate partly in real experiment and partly in simulations.

The method can be extended to general plant with considerable energy demands, where the decreasing actuator signal in a given range can preserve the system stability.

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