Active Suspension System – Energy Control

Katerina Hyniova*. Antonin Stribrsky*. Jaroslav Honcu*. Ales Kruczek*

*Department of Control Engineering, Faculty of Electrical Engineering, CTU Prague
Karlovo namesti 13, Prague 2, 12135, CZECH REPUBLIC
(e-mail: hyniova@fel.cvut.cz, stribrsk@fel.cvut.cz, honcu@fel.cvut.cz, kruczea@fel.cvut.cz)

Abstract: Suspension system is an important part of the car design, because it influences both the comfort and safety of the passengers. In this paper an active suspension using linear electric motor is designed. This article is also focused on experiments with active suspension – developing an appropriate input signal for the test bed and evaluation of the results. Second important aspect of the active suspension design is energy demand of the system. Modification of the standard controller which allows changing amount of energy required by the system has been designed. Performance of the modification was verified taking various experiments.

Keywords: robust control, active vehicle suspension, linear motors, energy control.

1. INTRODUCTION

In most active suspension systems, the biggest disadvantage consists in energy demands. Regarding linear electric motors, this drawback can be eliminated because under certain circumstances there is a possibility to recuperate energy, accumulate it and use it later when necessary. This way, it is possible to reduce or even eliminate the demands concerning the external power source. In the next chapters also the proposed strategy how to control the energy distribution is going to be described. In order to regenerate electric power from the vibrations excited by road unevenness a new energy-regenerative active suspension for vehicles has been proposed, then the active system has been modelled and simulated to show the performance improvement and the performance tests of the actuator prototype testing stand have been carried out.

All suspension systems are designed to meet specific requirements. In suspension systems, usually two most important features are expected to be improved - disturbance absorbing (i.e. passenger comfort) and attenuation of the disturbance transfer to the road (i.e. car handling). The first requirement could be presented as an attenuation of the damped mass acceleration or as a peak minimization of the damped mass vertical displacement. The second one is characterized as an attenuation of the force acting on the road or - in simple car model - as an attenuation of the unsprung mass acceleration. It is obvious that there is a contradiction between these two requirements. With respect to these contradictory requirements the best results can be achieved using active suspension systems generating variable mechanical force acting in the system using a linear electrical motor as the actuator. The electromagnetic force is applied directly without the intervention of a mechanical transmission. Low friction and no backlash resulting in high accuracy, high acceleration and velocity, high force, high reliability and long lifetime enable not only effective usage of modern control systems but also represent the important attributes needed to control vibration suspension efficiently.

2. THE SUSPENSION MODEL AND TEST BED

A simple model has been developed for controller design and simulations. The simplest car model is a one-quarter-car model. The basic schema is presented in Fig.1.

![Fig. 1. One-quarter-car model.](image)

The motion equations for the one-quarter-car model illustrated in the Fig.1 can be derived to describe the system. Therefore the equations are, in Kruczek et al. (2004):

\[ m_b \ddot{z}_b = F_a - k_b(z_b - z_w) - c_t(\dot{z}_b - \dot{z}_w) \]  

\[ m_w \ddot{z}_w = -F_a + k_b(z_b - z_w) + c_t(\dot{z}_b - \dot{z}_w) - k_s(z_w - z_r) \]
where:

- $z_r$ . . . road vertical displacement,
- $z_w$ . . . axle vertical displacement,
- $z_b$ . . . car body vertical displacement,
- $F_a$ . . . power of linear power source,
- $m_b$ . . . body mass = 250kg,
- $m_w$ . . . wheel mass and unsprung mass = 35kg,
- $c_1$ . . . damping constant = 1160Nsm$^{-1}$,
- $k_1$ . . . stiffness constant of spring = 15kNm$^{-1}$,
- $k_2$ . . . stiffness constant of tires = 115kNm$^{-1}$.

2.1 State-space model

The state and input variables has been chosen as follows:

- $x_1 = z_b - z_w$
- $x_2 = z_w - z_r$
- $x_3 = \dot{z}_b$
- $x_4 = \dot{z}_w$
- $u_1 = \dot{z}_r$
- $u_2 = F_a$

Then the conversion from the differential equations to the state-space matrices (2) is as follows:

$$\dot{x} = Ax + Bu_1 + B_2u_2$$ (2)

2.2 Simulations

Simulations have been performed for a one-quarter car model, a half car model and a full car model. Because of the one-quarter car character of the test bed, only one-quarter car model behavior could be verified.

The simple full-car model is shown in Fig.2.

![Fig. 2. Simple full-car model.](image)

A power source of displacement has generated the input signal i.e. road deviations using experimental signal described in next paragraphs. Mechanical configuration of the test bed is obvious in Fig.3.

![Fig. 3. Test bed.](image)

3. LINEAR ELECTRIC MOTOR

Linear electric motor has been used as an actuator generating required forces. Fig.4 represents the basic principle and configuration of the linear motor. The beauty of linear motors is that they directly translate electrical energy into usable linear mechanical force and motion, and vice versa. The motors are produced in synchronous and asynchronous versions. Compared to conventional rotational electro motors, the stator and the shaft (translator) of direct-drive linear motors are linear-shaped. One can imagine such a motor taking infinite stator diameter.

![Fig. 4. Linear motor - basic design (adapted manufacturer spreadsheet).](image)
Linear motor translator movements take place with high velocities (up to approximately 200 m/min), large accelerations (up to g multiples), and forces (up to kN). As mentioned above, the electromagnetic force can be applied directly to the payload without the intervention of a mechanical transmission, what results in high rigidity of the whole system, its higher reliability and longer lifetime. In practice, the most often used type is the synchronous three-phase linear motor.

It is necessary to answer one important question - if it is more advantageous to include the model of the linear electric motor in the model for active suspension synthesis or if it should be used only for simulations.

Comparing advantages and disadvantages of the model inclusion, it can be said that the closed-loop provides more information so that better control results can be achieved. Unfortunately, there are also some significant disadvantages in such a solution. The first one insists in the rank of the system (and consequently the rank of the controller which increases up to 5) and the second one is that the D matrix in the state space description of the motor model does not have full rank and that is why implementation functions are limited or too complicated. On the base of this comparison the linear motor has not been included in the model for active suspension synthesis.

There is another important question whether the linear motor model could be omitted and a linear character of the desired force could be supposed. The answer is “yes”. Both the mechanical and the electrical constants are very small – just about 1 ms. Moreover it will be shown that the robustness of the \( H_{\infty} \) control design has been verified using numerous simulation results and experiments.

4. EXPERIMENTAL SIGNAL

The most important step of the experiment design is the choice of a proper experimental signal representing a real road surface. Although the simplified suspension model seems to be linear the truth is that there exist many nonlinear parts. It implies the results will depend on inputs.

Now the question is how to choose the appropriate signal for experimental testing. We should define two types of input signals to conclude two objectives:

- to prove results of simulations and pre-calculations
- to test real behavior on the road

Let’s begin with the first – to verify simulation results. The best signal is probably white noise, because we can observe full frequency spectrum. But it should be noted it is a nonlinear system and even white noise is not sufficient. Moreover it is not possible to generate easily white noise on the test bed because there is none analytical expression of the signal.

A bump has been chosen as a signal which can be generated by the test bed. This signal allows observing both directions – up and down. It is not possible to generate infinite slope and during experiments the following signal has been used (Eq. 3). Different magnitudes of signal should be tested, because the system is nonlinear. Magnitudes has been chosen according to mechanical dimensions of the suspension system:

\[
\dot{z}_t(t) = 0.5\pi\sin20\pi(t - 0.1) \tag{3}
\]

Thus this signal is used to verify the quality of the simulation model, but does not give the usability of the controller at a real road situation.

More important results will be obtained using an input signal which is similar to the real road behavior. The random signal has been used to simulate it. The input signal for simulation can be described by the following equation:

\[
z_r(t) = \sum_{i=1}^{n} \sqrt{\frac{\omega_i}{\pi \cdot v_x}} \left\{ \frac{b_o}{-\omega^2 + a_1j\omega + a_o} \cdot \cos(\omega t + \alpha_i) + \frac{b_o}{-\omega^2 + a_1j\omega + a_o} \cdot \sin(\omega t + \alpha_i) \right\}
\]

\[
b_o = 0.121 \cdot v_x \\
a_o = 2.249 \cdot v_x \\
a_1 = 30.36 \cdot v_x
\]

where \( v_x \) in Eq.2 represents the car velocity and \( \alpha_i \) are randomly generated angles.

Thus resulted signal is obtained as a superposition of the sinusoids with deterministic “random”. In this case 128 random angles have been calculated. The used signal is plotted in Fig.6.

![Fig. 5. Recuperation range in linear motor.](image)
5. CONTROLLER

The controller for active suspension we have designed using $H_\infty$ theory. The standard $H_\infty$ control scheme is shown in Fig. 7. In this case the controller is robust enough when system parameters vary in a wide range. When the open loop transfer matrix from $u_1$ to $y_1$ is denoted as $T_{y_1u_1}$, then the standard optimal $H_\infty$ controller problem is to find all admissible controllers $K(s)$ such that $\|T_{y_1u_1}\|_\infty$ is minimal, where $\|\cdot\|_\infty$ denotes the $H_\infty$-norm of the transfer function (matrix). For more information, see Zhou et al. (1998).

The $H_\infty$ controller is stated minimizing the $\|T_{y_1u_1}\|$-norm. In addition, it is possible to shape open loop characteristics to improve performance of the whole system.

For the active suspension system the performance and robustness outputs should be weighted. The performance weighting has to include all significant measures as comfort and car stability (body speed, suspension displacement, actuator force, etc). For the linear electric motor in the position of an actuator, an additional weight should be added to control maximum force, energy consumption and robustness of the system.

5.1 Plant augmentation

The schematic diagram of a plant augmentation is shown in the Fig. 8. The first input signal $v$ represent the road disturbance and is scaled by factor $S_v$. The first group (port) includes the input $f$ too, which represents a disturbance signal acting on the measured feedback. Then the second input is desired value for the actuator. And as for outputs, the first output $y_1$ consists of two parts:

1. the nominal system output (states in our case) weighted by MIMO function $W_{per}$ and scaled by constant matrix $S_y$ and

2. the actuating signal weighted by $W_{rob}$. The second output is formed as a sum of nominal output $y_2$ and disturbance input $f$.

As for robustness function $W_{rob}$, to avoid some singularity problems in zero of nominal transfer function from $u_2$ to $y_2$ the additive uncertainty has been used. Whereas the function $W_{rob}$ doesn’t mean the conventional robustness stability function, but $W_{rob}$ is supposed to be total weighting by reason of linear motor nonlinearities, feedback branch disturbances and other uncertainties.

At first, it is necessary to choose the weighting functions, then what kind of feedback will be established. In this design output feedback has been used. Suspension speed has been chosen as a measured output ($y_2$), because for a linear motor control the speed has to be measured as well.

Moreover signals, which affect the important characteristics, have to be weighted in controlled output ($y_1$). Influence of weighting functions and constants is mainly evident, moreover it has been checked by simulations. So the first output is as follows:

1. $x_1 = \dot{z}_b - z_w$ ... weighted by constant, improve steady-state deviation value.

2. $x_2 = z_w - z_i$ ... weighted by constant, improve process of deviation stabilization.

3. $x_3 = \ddot{z}_d$ ... weighted by function, shape the sprung mass speed. It is a function ($W_{per}$), because the different shaping at each frequency is needed according to the human sensibility. This weighting imply the car comfort improvement character.
(4) \( x^4 \equiv \ddot{z}_w \ldots \) weighted by function, shape the unsprung mass acceleration. It is a function \( (W_{\text{per} f_2}) \), because this variable influences the vehicle stability and thus there should be the possibility to change the vehicle properties in the frequency domain.

(5) \( W_{\text{rob}} \ldots \) finally the function \( W_{\text{rob}} \) is weighted by constant to find the optimal value for car requirements satisfaction.

5.2 Weighting functions

Weighting function \( W_{\text{per} f_1} \) shapes the attenuation of the sprung mass acceleration. Human being is most sensitive in the frequency range between 4 and 8Hz, thus between 25 and 50rad/s. Then function \( W_{\text{per} f_1} \) has to be a band-pass filter. Complementary weighting function \( 1/W_{\text{per} f_1} \) is shown in Fig.9, where small correction at frequency 7rad/s can be seen. This correction has been obtained by trial and error method to achieve the best acceleration shaping.

Fig. 9. Weighting function \( 1/W_{\text{per} f_1} \).

After simulation tests, weighting function \( W_{\text{per} f_2} \) had to be considered constant.

Last weighting function is \( W_{\text{rob}} \). This function should, as has been mentioned above, respect the motor non-linearity and some inaccuracies and disturbances in the feedback loop. Complementary function \( 1/W_{\text{rob}} \) is plotted in Fig. 10.

Fig. 10. Weighting function \( 1/W_{\text{rob}} \).

6. QUANTIFICATION

Some quantitative measures have to be defined to evaluate the results achieved by the closed loop system and to compare the active and passive systems.

6.1 Car stability

First requirement in the active suspension system is to improve car stability and “road friendliness”, that can be characterized as the attenuation of the tire pressure, or more precisely the attenuation of the unsprung mass force acting on the road. To get a measurable parameter, the following RMS function can be introduced:

\[
J_{\text{stab}} = \sqrt{\int_0^T \left( \dot{z}_w - \dot{z}_r \right)^2 dt}
\]

where \( \dot{z}_w \) represents wheel displacement and \( \dot{z}_r \) road displacement.

6.2 Passenger comfort

Second important requirement in the active suspension system is to improve passenger comfort. This requirement can be formulated as the sprung mass acceleration attenuation when the RMS function is defined as:

\[
J_{\text{comf}} = \sqrt{\int_0^T G_w \ast \ddot{z}_b^2 dt}
\]

where \( \ddot{z}_b \) represents body acceleration, \( G_w \) is a weighting function for human sensitivity to vibrations and \( \ast \) denotes convolution.
7. ENERGY CONTROL

7.1 Energy control principles

The $H_\infty$ controller has been designed using appropriate weights to optimize minimum of the energy consumption with respect to the performance.

In the car, where the working conditions change according to the various drive situations it is very difficult (if possible) to say in general what level of performance is sufficient enough and how much energy can be obtained. It would be optimal to find a possibility of real-time control of the energy consumption. The energy management is supposed to be controlled by an external signal depending on the car and road parameters, i.e. on the energy accumulator capacity and on the road surface, respectively.

First possibility consists in the analysis of the driving conditions and cyclic re-computing of the control signal in real-time. While the time requirements of the $H_\infty$ controller design are too high (sampling period has to be less than 1ms!) and moreover the performance of the $H_\infty$ controller cannot be guaranteed for all operating conditions this approach has been rejected.

The second possibility is to control the energy consumption by controller deterioration. Then the designed $H_\infty$ controller is reliably robust and the active suspension system is relatively stable.

Let’s assume two driving conditions:

• the car is driving on is very rough and uneven and there is enough energy stored in the accumulator system - then the controller works in the standard mode, the motor consumes energy from the accumulator and the suspension performance is preserved.

• the car is driving on is relatively smooth and there is not enough energy stored in the accumulator system because of the situation described above. The external signal provides the information to the $H_\infty$ controller to deteriorate its performance and to reduce the energy consumption. The deterioration is stated by the desired force attenuation.

If the force is attenuated too much then the active suspension system works similarly to the passive suspension and the linear electric motor works as a generator producing energy for the accumulator system. Of course, the suspension performance is deteriorated now (to the passive suspension level in the worst case). The influence of these controller modifications to the suspension system performance we will be discussed in the section below.

The principle of the proposed energy management strategy is illustrated in Fig.11. The $H_\infty$ controller is extended by the variable gain block controlled by the external signal (energy management control input), in Stribrsky et al. (2007).

8. RESULTS

The random signal described above has been used for experiments on the test bed. Two important things should be supervised during experiments – suspension comfort improvement and energy consumption.

At first the body (sprung) mass displacement is plotted as an indicator of the comfort improvement. The influence of these controller modifications to the suspension system performance we will be discussed in the section below.

Fig. 11. Energy control schema.

The entire active suspension system is nonlinear. The output signal always depends on the input signal i.e. on the road surface. The road surface influences not only quality parameters of control (i.e. passenger comfort and stability) but also energy consumption in the system. Therefore the second control approaches is to design a set of individual controllers, when each of them is designed for a certain road surface. The individual controllers are switched by a supreme controller at a proper time point. The basic schema of this idea is shown in Fig. 12. This way of control has been only simulated these days but not verified using the test bed.

Fig. 12. Basic diagram for switching controllers.

8. RESULTS

The random signal described above has been used for experiments on the test bed. Two important things should be supervised during experiments – suspension comfort improvement and energy consumption.

At first the body (sprung) mass displacement is plotted as an indicator of the comfort improvement. There are three curves in Fig. 13 – the input signal (road displacement) and body displacement for two levels of the energy demands.
Table 2. Power mean values

<table>
<thead>
<tr>
<th></th>
<th>Mean value</th>
<th>Percentages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comfort setting</td>
<td>2.689</td>
<td>100%</td>
</tr>
<tr>
<td>Energy setting</td>
<td>1.598</td>
<td>59%</td>
</tr>
</tbody>
</table>

9. CONCLUSION

In this paper the $H_{\infty}$ controller for active suspension with linear electric motor has been used for experiment on the test bed. The experiment signal for real road simulation has been developed and then it has been used for experiments. The method for the direct real-time energy control with respect to reduction of the energy consumption has been used. Experiments verified validation of the simulations and showed that it is possible to change energy demands according to the road situation and status of the energy storage in the car (battery or super-capacitor). The method can be extended to general plants with considerable energy demands, where the decreasing actuator signal in a given range can preserve the system stability. Thus this controller with linear motor as an actuator can be used in any suspension system.

ACKNOWLEDGEMENT

This research has been supported by the MSMT project No. 1M6840770002 “Josef Bozek’s Research Center of Combustion Engines and Automobiles II” and MSMT projects INGO No. LA296 and No. LA299.

REFERENCES


Let’s show the results as mean values. Table 1 involves mean values for the road and body displacements as absolute values and as a percentage of the improvement.

Table 1. Displacement mean values

<table>
<thead>
<tr>
<th></th>
<th>Mean value</th>
<th>Percentages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body displacement-comfort</td>
<td>38.6</td>
<td>100%</td>
</tr>
<tr>
<td>Body displacement-energy</td>
<td>47.4</td>
<td>123%</td>
</tr>
</tbody>
</table>

Table 2 involves mean values of energy. Lower value corresponds the lower energy demand.